

# A Survey on Space-Vector Pulse Width Modulation for Multilevel Inverters

Qamar Muhammad Attique, Yongdong Li, and Kui Wang

**Abstract**—The selection of an appropriate modulation scheme plays a vital role to assure the performance of multilevel inverters. Space vector pulse width modulation (SVPWM) is more efficient among all other pulse width modulation (PWM) techniques due to its key characteristics like better DC voltage utilization, switching losses reduction and easiness in digital implementation. The conventional SVPWM scheme presents some computational complexities due to redundant switching states and large number of space vectors. This paper summarizes five different SVPWM techniques for multilevel inverters which are  $\alpha$ - $\beta$  frame,  $g$ - $h$  frame,  $K$ - $L$  frame,  $\alpha'$ - $\beta'$  frame and SVPWM based on imaginary coordinate system.  $g$ - $h$  frame and  $K$ - $L$  frame are based on  $60^\circ$  and  $120^\circ$  coordinates system respectively. To compare the result of these SVPWM schemes, the complex calculations of conventional SVPWM are converted into simplified line voltages form. The comparison results validate all the SVPWM techniques, but the SVPWM based on imaginary coordinate is found more simple in duty ratio calculations, easier to understand and provides a better control for zero-sequence component for any level of inverter.

**Index Terms**— $g$ - $h$  frame, imaginary coordinate system,  $K$ - $L$  frame, multilevel inverters, space vector pulse width modulation (SVPWM),  $\alpha'$ - $\beta'$  frame.

## I. INTRODUCTION

NOWADAYS, the extensive use of multilevel inverters in high power and high voltage applications has made it a point of attraction for the researchers because of their remarkable performance. As compared with two-level inverters, multilevel inverters have various advantages, e.g. less harmonics in output current and voltages, reduced voltage stress across switching devices, lower  $dv/dt$ , better output wave form quality and lower common mode voltages [1], [2]. The diode-clamped 3 level neutral-point clamped (NPC) topology, as shown in Fig. 1 has been the most widely used one among all multilevel inverter topologies that have been proposed in literature [3]-[5].

By using an appropriate PWM technique, from discrete voltage levels, multilevel inverters generate the sinusoidal output voltages of different frequencies. Multilevel inverter's output performance depends on modulation algorithm and various PWM algorithms have been developed so far to fulfill the following objectives: less total harmonic distortion (THD), wider linear modulation range, lower switching losses and easy

implementation. Among these, the two most popular PWM generation algorithms for multilevel inverters are sinusoidal carrier-based PWM (SPWM) and space vector PWM (SVPWM). In engineering applications, SPWM algorithms maintained their credibility for a long period [6]-[10] but with the development of microcontrollers, SVPWM took place due to its easy digital implementation, better harmonics performance, high DC voltage utilization ratio, reduced switching losses and convenience for capacitor voltage balancing. Moreover, the SVPWM has 15% higher linear modulation range than that of SPWM [11]. However, by increasing the number of levels, SVPWM faces the problem of more complex computations as compared to carrier-based PWM. Therefore many efforts have been made to achieve the SVPWM's performance by using zero-sequence voltage injection in carrier-based PWM [12]-[16]. Relationship between space vector and phase disposition carrier modulation is presented in [17] for hybrid and diode clamped multilevel inverters.

Any  $N$ -level inverter consists of six sectors and  $N^3$  switching states in its space vector (SV) diagram while each sector comprises on  $(N-1)^2$  triangles. There are  $1 + s \sum_{i=1}^{N-1} i$  (here  $S$  is the total number of sectors and  $N$  shows number of level) switching vectors having one or more switching states that depends on its location in space vector diagram. The switching states with equal line to line voltages are known as redundant states in SV diagram. By choosing an optimal switching sequence of these states, certain objectives can be accomplished such as: common-mode voltage reduction [18]-[20], extension of modulation index [21], fault tolerance operation [22], switching frequency reduction [20], [23] and balancing of DC link capacitor voltages [18], [24]-[27]. Voltage balancing of DC-link capacitors in NPC converters is one of the essential problems that causes the deviation of output voltages from the reference value, also damage the equipment and devices [28]-[33]. If the sinusoidal PWM algorithm is used, the control of neutral point voltage is done by injecting the appropriate zero-sequence voltage in the reference voltage [34]-[37]. A relation between zero-sequence voltage and neutral current for NPC three-level converters is presented in [38].

By increasing the level of inverter, number of switching states and triangles becomes quite large which causes complexities in on-time calculations of switching periods. For example, the space vector diagram for three-level inverter has 27 switching states and 24 triangles while five-level inverter has 125 switching states and 96 triangles. So, the traditional SVPWM algorithms [39], [40] by using trigonometric function calculations becomes impractical as level of inverters increases. So far, various SVPWM algorithms have been presented in literature [41]-

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[51] to diminish the problem of computational complexities.

A decomposition method was introduced in [41] and also implemented in [42], [43], by which the SV hexagon of three-level inverter is decomposed into six two-level SV hexagons. This method still has huge complications for increased level. For five-level inverter, SV diagram should be decomposed into six hexagons of four-level and then each four-level hexagon should be decomposed into six hexagons of three-level and so on. A fast SVPWM algorithm based on  $60^\circ$  coordinate system has been presented in [44] but does not consider proper switching states sequence and capacitor voltage balancing. Another effort has been made in [47] by proposing a general SVPWM algorithm based on imaginary coordinate system including the control of zero-sequence component. Two different non-orthogonal SVPWM strategies have been implemented in [48] and [51].

This paper summarizes the five different SVPWM algorithms for multilevel inverters which are  $\alpha$ - $\beta$  frame,  $g$ - $h$  frame,  $K$ - $L$  frame,  $\alpha'$ - $\beta'$  frame and SVPWM based on imaginary coordinate system. The complex calculations of conventional SVPWM are converted into simplified line voltages form and the results are compared for all aforementioned algorithms. The comparison results validate all the SVPWM techniques, but the SVPWM based on imaginary coordinate is more simple in duty ratio calculations, easier to understand and provides a better control for zero-sequence component for any level of inverter.

## II. THE CONVENTIONAL ( $\alpha$ - $\beta$ FRAME) SVPWM ALGORITHM

The basic diagram of 3-level diode-clamped inverter is shown in Fig. 1 which contains twelve switching devices (four in each leg) and three output levels for each phase (2, 1, 0). The relation between these devices and output level of each leg is shown in TABLE I. Fig. 2 shows the space vector diagram of three level inverter which consists of 24 active voltage vectors (6 large vectors, 6 medium vectors, 12 small vectors) and three zero vectors (222, 111, 000). SV diagram is divided into six sectors (labeled as sector I to VI), each sector is divided into four triangles (labeled as triangle 1 to 4), 24 triangles in total. The implementation of space vector PWM comprises of the sector identification where the reference voltage vector is located, identification of three nearest switching vectors, selection of an appropriate switching sequence and on-time calculation of switches for specific switching sequence.

The reference vector  $V_{ref}$  corresponding to the three-phase

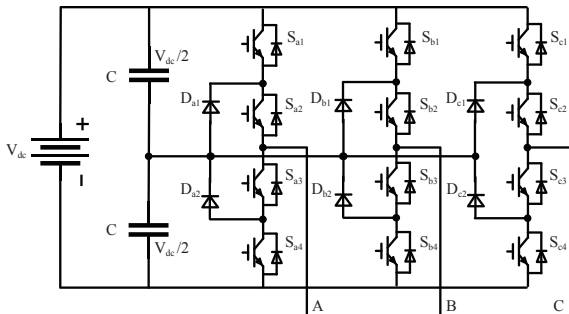


Fig. 1. Three level diode clamped NPC inverter.

TABLE I  
RELATIONSHIP BETWEEN SWITCHING DEVICES AND OUTPUT LEVEL

ON Devices	OFF Devices	Output Level	Terminal Voltage
$S_{x1}, S_{x2}$	$S_{x3}, S_{x4}$	2	$+V_{dc}/2$
$S_{x2}, S_{x3}$	$S_{x1}, S_{x4}$	1	0
$S_{x3}, S_{x4}$	$S_{x1}, S_{x2}$	0	$-V_{dc}/2$

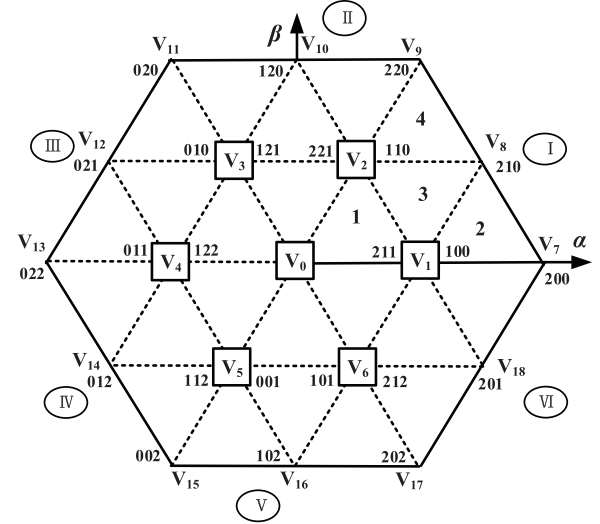


Fig. 2. Space vector diagram of three level NPC inverter.

voltages is defined by

$$V_{ref} = 2(V_a + V_b e^{j2\pi/3} + V_c e^{-j2\pi/3})/3. \quad (1)$$

By using Clark transformation, three-phase coordinate system  $a$ - $b$ - $c$  is transformed to 2-dimensional  $\alpha$ - $\beta$  frame which is helpful in sector identification by following expression:

$$\begin{bmatrix} V_{r\alpha} \\ V_{r\beta} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (2)$$

where  $V_{r\alpha}$  and  $V_{r\beta}$  are the components of reference vector in  $\alpha$ - $\beta$  coordinate system. The reference voltage vector can be located in any region (1-4) of any sector (I-VI) in space vector diagram as shown in Fig. 2. For example, considering that the reference vector  $V_{ref}$  is locating in region 2 of sector I, the nearest three voltage vectors for this region are  $V_1$ ,  $V_7$  and  $V_8$  are shown in Fig. 3.

After identification of nearest three vectors, the following expressions can be developed for the on-time calculations of the corresponding vectors by using the volt-second balance method

$$\begin{cases} V_{ref} T_s = V_8 t_a + V_1 t_b + V_7 t_c \\ T_s = t_a + t_b + t_c \end{cases} \quad (3)$$

where  $T_s$  is the sampling time,  $t_a$ ,  $t_b$  and  $t_c$  are the on-times for

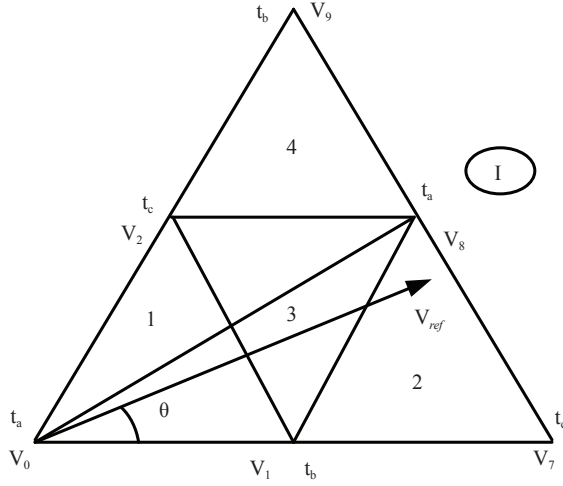


Fig. 3. Vector representation in sector I.

the voltage vectors  $V_8$ ,  $V_1$  and  $V_7$  respectively. By solving (3), the on-time of the corresponding vectors can be computed by

$$\left. \begin{aligned} t_a &= a \sin \theta / \sqrt{3} \\ t_b &= 2T_s - a \sin (\theta + \pi/3) / \sqrt{3} \\ t_c &= -T_s - a \sin (\theta - \pi/3) / \sqrt{3} \end{aligned} \right\} \quad (4)$$

where  $a=3T_s (V_{ref}/V_{dc})$ . The on-times for all the corresponding vectors in remaining regions of sector I can be computed by using similar procedure as expressed in TABLE II.

TABLE II  
ON-TIMES IN SECTOR I

	$t_a$	$t_b$	$t_c$
Region 1	$T_s - a \sin (\theta + \pi/3) / \sqrt{3}$	$-a \sin (\theta - \pi/3) / \sqrt{3}$	$a \sin \theta / \sqrt{3}$
Region 2	$a \sin \theta / \sqrt{3}$	$2T_s - a \sin (\theta + \pi/3) / \sqrt{3}$	$-T_s - a \sin (\theta - \pi/3) / \sqrt{3}$
Region 3	$-T_s + a \sin (\theta + \pi/3) / \sqrt{3}$	$T_s - a \sin \theta / \sqrt{3}$	$T_s + a \sin (\theta - \pi/3) / \sqrt{3}$
Region 4	$-a \sin (\theta - \pi/3) / \sqrt{3}$	$-T_s + a \sin \theta / \sqrt{3}$	$2T_s - a \sin (\theta + \pi/3) / \sqrt{3}$

The next step in implementation of SVPWM is the selection of optimal switching sequence of the redundant states which is helpful in balancing of DC link capacitor voltages, fault tolerance and switching frequency reduction etc. TABLE III shows all possible switching sequence for all regions of sector I and Fig. 4 is the graphical representation for region 2.

The conventional SVPWM requires a huge amount of trigonometric operations to calculate the on-times of the switches which needs more storage space and additional hardware. There are three types of trigonometric functions ( $\sin \theta$ ,  $\sin(\theta+\pi/3)$ ,  $\sin(\theta-\pi/3)$ ) in TABLE I. These trigonometric functions can be

TABLE III  
POSSIBLE SWITCHING SEQUENCE FOR SECTOR I

Region	Switching Sequence
1	(i) 222-221-211-111 (ii) 000-100-110-111 (iii) 221-211-111-110 (iv) 211-111-110-100
2	(i) 221-211-210-110 (ii) 211-210-110-100
3	221-220-210-110
4	211-210-200-100

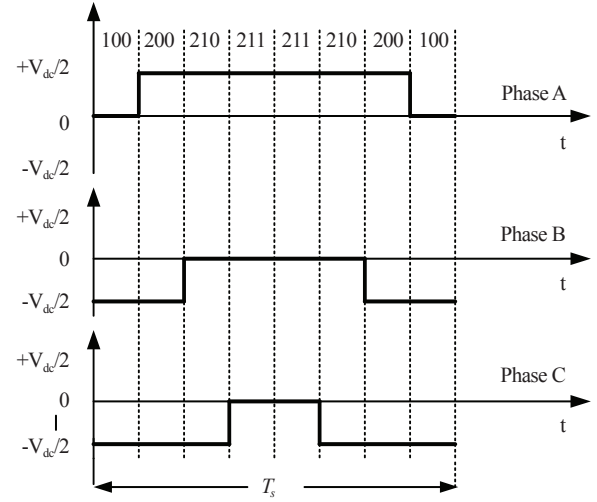


Fig. 4. Switching sequence for region 2.

converted into simplified line voltages form by using few calculations as follows:

$$|V_{ref}| \sin \theta = |V_{ref}| \sqrt{\tan^2 \theta / (1 + \tan^2 \theta)} \quad (5)$$

$$|V_{ref}| = \sqrt{\frac{2}{3} \left[ \left( V_a - \frac{1}{2} V_b - \frac{1}{2} V_c \right)^2 + \left( \frac{\sqrt{3}}{2} V_b - \frac{\sqrt{3}}{2} V_c \right)^2 \right]} \quad (6)$$

$$\tan \theta = \frac{\text{Re}(V_{ref})}{\text{Im}(V_{ref})} = \frac{\frac{\sqrt{3}}{2} V_b - \frac{\sqrt{3}}{2} V_c}{V_a - \frac{1}{2} V_b - \frac{1}{2} V_c} \quad (7)$$

By using (6) and (7) into (5), the result in simplified line voltages form is

$$|V_{ref}| \sin \theta = V_{bc} / \sqrt{3} \quad (8)$$

The remaining two trigonometric functions can be calculated by using same method and the results are shown in (9) and (10).

$$|V_{ref}| \sin (\theta + \pi/3) = -V_{ca} / \sqrt{3} \quad (9)$$

$$|V_{ref}| \sin (\theta - \pi/3) = -V_{ab} / \sqrt{3} \quad (10)$$

Now the complex computations of the conventional SVPWM algorithm are converted into simplified line voltages form by replacing the values of trigonometric functions as shown in TABLE IV.

TABLE IV  
ON-TIMES IN LINE VOLTAGES FORM FOR SECTOR I

	$t_a$	$t_b$	$t_c$
Region 1	$T_s \left(1 + \frac{V_{ca}}{V_{dc}}\right)$	$T_s \frac{V_{ab}}{V_{dc}}$	$T_s \frac{V_{bc}}{V_{dc}}$
Region 2	$T_s \frac{V_{bc}}{V_{dc}}$	$T_s \left(2 + \frac{V_{ca}}{V_{dc}}\right)$	$T_s \left(-1 + \frac{V_{ab}}{V_{dc}}\right)$
Region 3	$T_s \left(-1 - \frac{V_{ca}}{V_{dc}}\right)$	$T_s \left(1 - \frac{V_{bc}}{V_{dc}}\right)$	$T_s \left(1 - \frac{V_{ab}}{V_{dc}}\right)$
Region 4	$T_s \frac{V_{ab}}{V_{dc}}$	$T_s \left(-1 + \frac{V_{bc}}{V_{dc}}\right)$	$T_s \left(2 + \frac{V_{ca}}{V_{dc}}\right)$

### III. SVPWM BASED ON $g$ - $h$ COORDINATE SYSTEM

The space vector diagram for three-level inverter based on  $60^\circ$  coordinate system is shown in Fig. 5. The  $g$ -axis overlapped with  $\alpha$ -axis and the  $h$ -axis is  $60^\circ$  apart from  $g$ -axis in counter-clockwise direction.

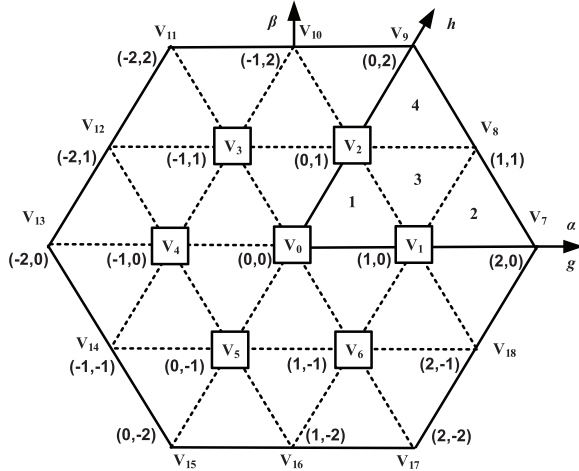


Fig. 5. Space vector diagram of inverter in  $g$ - $h$  frame.

#### A. Coordinate Transformation

The transformation of reference vector  $V_{ref}$  from three-phase system to two dimensional  $g$ - $h$  frame is the basic step of this algorithm. The relation between  $\alpha$ - $\beta$  coordinate system and  $g$ - $h$  coordinate system is given by

$$\begin{bmatrix} V_{r\alpha} \\ V_{r\beta} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 0 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_{rg} \\ V_{rh} \end{bmatrix} \quad (11)$$

and

$$\begin{bmatrix} V_{rg} \\ V_{rh} \end{bmatrix} = \begin{bmatrix} 1 & -1/\sqrt{3} \\ 0 & 2/\sqrt{3} \end{bmatrix} \begin{bmatrix} V_{r\alpha} \\ V_{r\beta} \end{bmatrix} \quad (12)$$

where  $V_{rg}$  and  $V_{rh}$  are the components of reference voltage vector  $V_{ref}$  in  $g$ - $h$  coordinate system. By using (2) and (12) the desired coordinate transformation can be achieved as shown in the following expression.

$$\begin{bmatrix} V_{rg} \\ V_{rh} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (13)$$

Fig. 6 shows that the lines  $l_g, l_{g+1}, l_{g+2}, \dots$  are parallel to the  $h$ -axis and the projections of every vector located on these lines are  $gV_{dc}, (g+1)V_{dc}, (g+2)V_{dc}, \dots$  respectively on the  $g$ -axis. Similarly the lines  $l_h, l_{h+1}, l_{h+2}, \dots$  are parallel to the  $g$ -axis and the projections of every vector located on these lines are  $hV_{dc}, (h+1)V_{dc}, (h+2)V_{dc}, \dots$  respectively on the  $h$ -axis.

#### B. Identification of Nearest Three Vectors

All the switching vectors in  $g$ - $h$  coordinate system have only integer coordinates, so the real values of the coordinates are rounded down to the integer values by using the  $\text{int}()$  function.

$$\begin{cases} g = \text{int}(V_{rg}) \\ h = \text{int}(V_{rh}) \end{cases} \quad (14)$$

The nearest four vectors constitute a rhombus which is divided into triangle I and triangle II as shown in Fig. 6. The coordinates of vectors  $V_a, V_b, V_c$  and  $V_d$  are  $(\text{int}(V_{rg}), \text{int}(V_{rh}))$ ,  $(\text{int}(V_{rg})+1, \text{int}(V_{rh}))$ ,  $(\text{int}(V_{rg}), \text{int}(V_{rh})+1)$ , and  $(\text{int}(V_{rg})+1, \text{int}(V_{rh})+1)$  respectively. The reference vector is locating in triangle I or triangle II, the decision can be made by the following expression:

$$\begin{cases} V_{ca}+1 \geq -(g+h) & V_{ref} \text{ in triangle I} \\ V_{ca}+1 \leq -(g+h) & V_{ref} \text{ in triangle II} \end{cases} \quad (15)$$

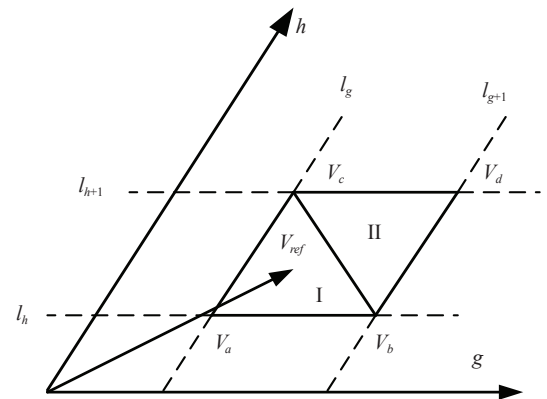


Fig. 6. The composition of space vector in  $g$ - $h$  frame.

### C. Duty Cycle Calculation

Assuming that the reference vector is locating in triangle I, the nearest three vectors are  $V_a$ ,  $V_b$  and  $V_c$ . The duty ratio of the corresponding vectors can be calculated by

$$\left. \begin{aligned} V_{ref}T_s &= V_a t_a + V_b t_b + V_c t_c \\ T_s &= t_a + t_b + t_c \end{aligned} \right\}. \quad (16)$$

Transforming (16) into  $g$ - $h$  coordinates yields

$$\left. \begin{aligned} V_{rg}T_s &= V_{ag}t_{g,h} + V_{bg}t_{g+1,h} + V_{cg}t_{g,h+1} \\ V_{rh}T_s &= V_{ah}t_{g,h} + V_{bh}t_{g+1,h} + V_{ch}t_{g,h+1} \\ T_s &= t_{g,h} + t_{g+1,h} + t_{g,h+1} \end{aligned} \right\}. \quad (17)$$

By solving (17), the on-time of the corresponding vectors can be computed by

$$\left. \begin{aligned} t_{g,h} &= T_s \left( (V_{ca}/V_{dc}) + g + h + 1 \right) \\ t_{g+1,h} &= T_s \left( (V_{ab}/V_{dc}) - g \right) \\ t_{g,h+1} &= T_s \left( (V_{bc}/V_{dc}) - h \right) \end{aligned} \right\}. \quad (18)$$

Similarly, when the reference vector is locating in triangle II, the on-times for the vectors  $V_b$ ,  $V_c$  and  $V_d$  can be calculated by

$$\left. \begin{aligned} t_{g+1,h} &= T_s \left( -(V_{bc}/V_{dc}) + g + 1 \right) \\ t_{g,h+1} &= T_s \left( -(V_{ab}/V_{dc}) + g + 1 \right) \\ t_{g+1,h+1} &= T_s \left( -(V_{ca}/V_{dc}) - g - h - 1 \right) \end{aligned} \right\}. \quad (19)$$

Assuming that, the amplitude of reference vector is  $1.386V_{dc}$  and it is  $25^\circ$  electrical with respect to the  $a$ -axis. The corresponding line voltages are

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = 1.386 \cdot V_{dc} \begin{bmatrix} 0.574 \\ 0.422 \\ -0.966 \end{bmatrix} \quad (20)$$

and the value of  $V_{rg}$  and  $V_{rh}$  is  $0.795V_{dc}$  and  $0.585V_{dc}$  respectively. The exact location of reference vector can be determined by using (15).

$$V_{ca} + 1 \leq -(g + h) = -0.380 < 0 \quad (21)$$

The reference vector is located in triangle 3 in Fig. 5 and the nearest three vectors are  $V_1(1, 0)$ ,  $V_2(0, 1)$  and  $V_8(1, 1)$ . The duty cycle for these vectors is calculated by (19).

$$\left. \begin{aligned} t_1 &= 0.415T_s \\ t_2 &= 0.205T_s \\ t_8 &= 0.380T_s \end{aligned} \right\} \quad (22)$$

### D. Switching-State Selection

The last step of the algorithm is to transform the two-dimen-

sional system back to three-dimensional switching states by using the following expression:

$$\begin{bmatrix} S_A \\ S_B \\ S_C \end{bmatrix} = \begin{bmatrix} i \\ i - g \\ i - h \end{bmatrix} \in (0, 1, 2), \quad i \in (0, 1, 2) \quad (23)$$

For example, the vectors (0, 1) and (1, 1) in the Fig. 5 can be transformed into (1 1 0), (2 2 1) and (2 1 0) switching states respectively.

## IV. SVPWM BASED ON $K$ - $L$ COORDINATE SYSTEM

The  $K$ - $L$  coordinate system is also called  $120^\circ$  coordinate system. Fig. 7 shows the space vector diagram for three-level inverter based on  $K$ - $L$  coordinate system. The  $L$ -axis coincided with  $\alpha$ -axis and the  $K$ -axis is  $120^\circ$  apart from  $L$ -axis in counter-clockwise direction.

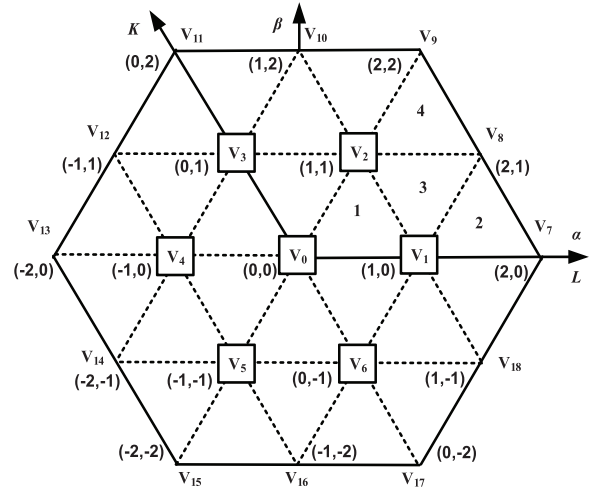


Fig. 7. Space vector diagram of inverter in  $K$ - $L$  frame.

### A. Coordinate Transformation

The initial step that is required in this algorithm is the transformation of reference vector  $V_{ref}$  from three-dimensional system to two dimensional  $K$ - $L$  coordinate system. The relation between  $\alpha$ - $\beta$  coordinate system and  $K$ - $L$  coordinate system is given by

$$\begin{bmatrix} V_{r\alpha} \\ V_{r\beta} \end{bmatrix} = \begin{bmatrix} 1 & -1/2 \\ 0 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_{rL} \\ V_{rK} \end{bmatrix} \quad (24)$$

and

$$\begin{bmatrix} V_{rL} \\ V_{rK} \end{bmatrix} = \begin{bmatrix} 1 & 1/\sqrt{3} \\ 0 & 2/\sqrt{3} \end{bmatrix} \begin{bmatrix} V_{r\alpha} \\ V_{r\beta} \end{bmatrix} \quad (25)$$

where  $V_{rL}$  and  $V_{rK}$  are the components of reference voltage vec-



tor  $V_{ref}$  in  $K$ - $L$  coordinate system while  $V_{ra}$  and  $V_{rk}$  are the components of  $V_{ref}$  in  $\alpha$ - $\beta$  coordinate system. By using (2) and (25) the desired coordinate transformation can be achieved as shown in the following expression:

$$\begin{bmatrix} V_{rL} \\ V_{rK} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}. \quad (26)$$

The lines  $l_L, l_{L+1}, l_{L+2}, \dots$  are parallel to the  $K$ -axis and the components of every vector located on these lines are  $LV_{dc}, (L+1)V_{dc}, (L+2)V_{dc}, \dots$  respectively on the  $L$ -axis as shown in Fig. 8. Similarly the lines  $l_K, l_{K+1}, l_{K+2}, \dots$  are parallel to the  $L$ -axis and the components of every vector located on these lines are  $KV_{dc}, (K+1)V_{dc}, (K+2)V_{dc}, \dots$  respectively on the  $K$ -axis.

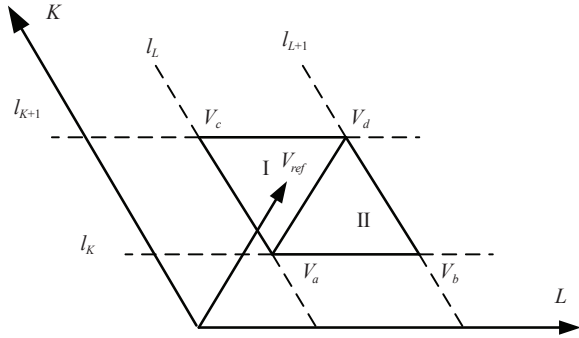


Fig. 8. The composition of space vector in  $K$ - $L$  frame.

### B. Identification of Nearest Three Vectors

As shown in Fig. 7, all the switching vectors in  $K$ - $L$  coordinate system have only integer coordinates, so the real values of the coordinates are rounded down to the integer values by using the  $\text{int}()$  function.

$$\begin{aligned} L &= \text{int}(V_{rL}) \\ K &= \text{int}(V_{rK}) \end{aligned} \quad (27)$$

The coordinates of the nearest four vectors  $V_a, V_b, V_c$  and  $V_d$  are  $(\text{int}(V_{rL}), \text{int}(V_{rK}))$ ,  $(\text{int}(V_{rL})+1, \text{int}(V_{rK}))$ ,  $(\text{int}(V_{rL}), \text{int}(V_{rK})+1)$  and  $(\text{int}(V_{rL})+1, \text{int}(V_{rK})+1)$  respectively. Whether the reference vector is locating in triangle I or triangle II, the decision can be made by the following expression:

$$\begin{cases} V_{ab} \leq L - K & V_{ref} \text{ in triangle I} \\ V_{ab} \geq L - K & V_{ref} \text{ in triangle II} \end{cases} \quad (28)$$

### C. Duty Cycle Calculation

Assuming that the reference vector is locating in triangle I, the nearest three vectors are  $V_a, V_c$  and  $V_d$ . The duty ratio of the corresponding vectors can be calculated by

$$\begin{cases} V_{ref}T_s = V_a t_a + V_c t_c + V_d t_d \\ T_s = t_a + t_c + t_d \end{cases} \quad (29)$$

Transforming (29) into  $K$ - $L$  coordinates yields

$$\begin{cases} V_{rL}T_s = V_a t_{L,K} + V_c t_{L,K+1} + V_d t_{L+1,K+1} \\ V_{rK}T_s = V_a t_{L,K} + V_c t_{L,K+1} + V_d t_{L+1,K+1} \\ T_s = t_{L,K} + t_{L,K+1} + t_{L+1,K+1} \end{cases} \quad (30)$$

By solving (30), the on-time of the corresponding vectors can be computed by

$$\begin{cases} t_{L,K} = T_s \left( -(V_{bc}/V_{dc}) + K + 1 \right) \\ t_{L,K+1} = T_s \left( -(V_{ab}/V_{dc}) + L - K \right) \\ t_{L+1,K+1} = T_s \left( -(V_{ca}/V_{dc}) - L \right) \end{cases} \quad (31)$$

Similarly, when the reference vector is locating in triangle II, the on-times for the vectors  $V_a, V_b$  and  $V_d$  can be calculated by

$$\begin{cases} t_{L,K} = T_s \left( (V_{ca}/V_{dc}) + L + 1 \right) \\ t_{L+1,K} = T_s \left( (V_{ab}/V_{dc}) - L + K \right) \\ t_{L+1,K+1} = T_s \left( (V_{bc}/V_{dc}) - K \right) \end{cases} \quad (32)$$

For example, by keeping the amplitude and angle of the reference vector same as  $g$ - $h$  coordinate system, the value of  $V_{rL}$  and  $V_{rK}$  is  $1.380V_{dc}$  and  $0.585V_{dc}$  respectively. The exact location of reference vector can be determined by using (28).

$$V_{ab} \leq L - K = 0.795 < 1 \quad (33)$$

The reference vector is located in triangle 3 in Fig. 7 and the nearest three vectors are  $V_1(1, 0)$ ,  $V_2(0, 1)$  and  $V_8(1, 1)$ . The duty cycle for these vectors is calculated by (31).

$$\begin{cases} t_1 = 0.415T_s \\ t_2 = 0.205T_s \\ t_8 = 0.380T_s \end{cases} \quad (34)$$

### D. Switching-State Selection

The last step of the algorithm is to transform the two-dimensional system back to three-dimensional switching states by using the following expression.

$$\begin{bmatrix} S_A \\ S_B \\ S_C \end{bmatrix} = \begin{bmatrix} i + L \\ i + K \\ i \end{bmatrix} \in (0, 1, 2), \quad i \in (0, 1, 2) \quad (35)$$

For example, the vectors  $(1, 0)$  and  $(2, 0)$  in the Fig. 7 can be transformed into  $(1 \ 0 \ 0)$ ,  $(2 \ 1 \ 1)$  and  $(2 \ 0 \ 0)$  switching states

respectively.

### V. SVPWM BASED ON $\alpha'$ - $\beta'$ COORDINATE SYSTEM

The conventional SVPWM algorithm is based on  $\alpha$ - $\beta$  coordinate system and reference vector rotates in a circular trajectory. In this algorithm, circular trajectory of reference vector is transformed into an elliptical trajectory in  $\alpha'$ - $\beta'$  frame as shown in Fig. 9.

#### A. Coordinate Transformation

The relationship between  $\alpha$ - $\beta$  frame and  $\alpha'$ - $\beta'$  frame is given by

$$\begin{bmatrix} V_{r\alpha} \\ V_{r\beta} \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ \sqrt{3}/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_{r\alpha'} \\ V_{r\beta'} \end{bmatrix} \quad (36)$$

and

$$\begin{bmatrix} V_{r\alpha'} \\ V_{r\beta'} \end{bmatrix} = \begin{bmatrix} 1 & 1/\sqrt{3} \\ -1 & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} V_{r\alpha} \\ V_{r\beta} \end{bmatrix} \quad (37)$$

where  $V_{r\alpha}$  and  $V_{r\beta}$  are the components of reference voltage vector  $V_{ref}$  in  $\alpha$ - $\beta$  coordinate system. The coordinate transformation from three-phase system to  $\alpha'$ - $\beta'$  frame can be achieved by using (2) and (37) as shown in (38).

$$\begin{bmatrix} V_{r\alpha'} \\ V_{r\beta'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (38)$$

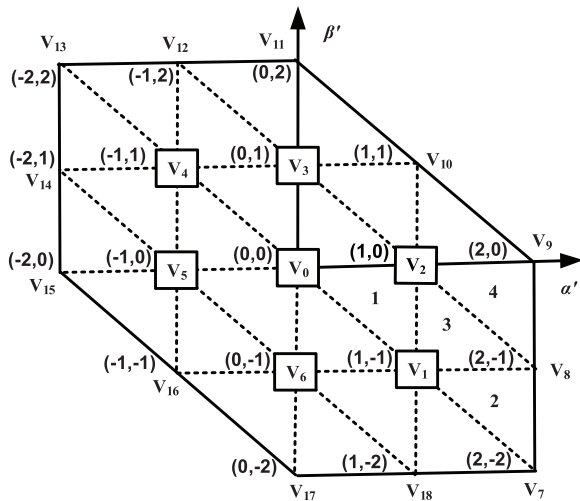


Fig. 9. Space vector diagram of inverter in  $\alpha'$ - $\beta'$  frame.

#### B. Identification of Nearest Three Vectors

As shown in Fig. 10, the vertices of the vectors  $V_a$ ,  $V_b$  and  $V_c$

constitute triangles I and triangle II is made by the vertices of the vectors  $V_b$ ,  $V_c$  and  $V_d$ . The coordinates of the nearest four vectors  $V_a$ ,  $V_b$ ,  $V_c$  and  $V_d$  are  $(int(V_{r\alpha}), int(V_{r\beta}))$ ,  $(int(V_{r\alpha})+1, int(V_{r\beta}))$ ,  $(int(V_{r\alpha}), int(V_{r\beta})+1)$  and  $(int(V_{r\alpha})+1, int(V_{r\beta})+1)$  respectively.  $int()$  is a function that is used to round down the real values to the integer values of the coordinates.

$$\begin{cases} \alpha' = int(V_{r\alpha'}) \\ \beta' = int(V_{r\beta'}) \end{cases} \quad (39)$$

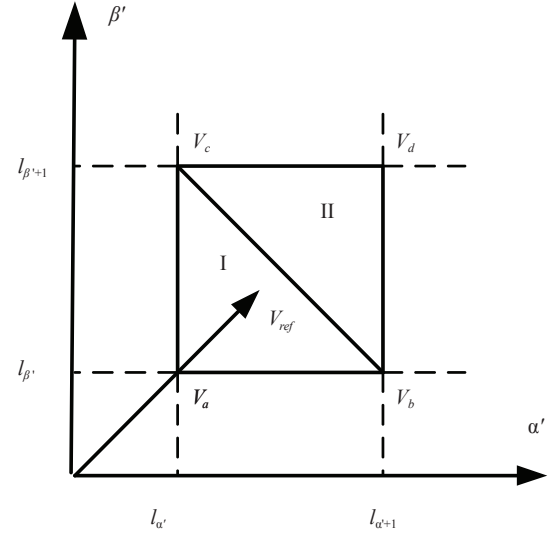


Fig. 10. The composition of space vector in  $\alpha'$ - $\beta'$  frame.

Vectors  $V_b$  and  $V_c$  are always the nearest two vectors while the third nearest vector will be decided by the following expression:

$$W = V_{bc} - \alpha' - \beta' - 1. \quad (40)$$

If  $W < 0$ , the third nearest vector will be  $V_a$  and  $V_{ref}$  will be located in triangle I; otherwise, reference vector will be in triangle II.

#### C. Duty Cycle Calculation

Assuming that the reference vector is locating in triangle I, the nearest three vectors are  $V_a$ ,  $V_b$  and  $V_c$ . The duty ratio of the corresponding vectors can be calculated by

$$\begin{cases} V_{ref}T_s = V_a t_a + V_b t_b + V_c t_c \\ T_s = t_a + t_b + t_c \end{cases} \quad (41)$$

Transforming (41) into  $\alpha'$ - $\beta'$  coordinates yields

$$\begin{cases} V_{r\alpha'}T_s = V_{a\alpha'}t_{\alpha',\beta'} + V_{b\alpha'}t_{\alpha'+1,\beta'} + V_{c\alpha'}t_{\alpha',\beta'+1} \\ V_{r\beta'}T_s = V_{a\beta'}t_{\alpha',\beta'} + V_{b\beta'}t_{\alpha'+1,\beta'} + V_{c\beta'}t_{\alpha',\beta'+1} \\ T_s = t_{\alpha',\beta'} + t_{\alpha'+1,\beta'} + t_{\alpha',\beta'+1} \end{cases} \quad (42)$$

By solving (42), the on-time of the corresponding vectors can

be computed by

$$\left. \begin{aligned} t_{\alpha',\beta'} &= T_s \left( -(V_{bc}/V_{dc}) + \alpha' + \beta' + 1 \right) \\ t_{\alpha'+1,\beta'} &= T_s \left( -(V_{ca}/V_{dc}) - \alpha' \right) \\ t_{\alpha',\beta'+1} &= T_s \left( -(V_{ab}/V_{dc}) - \beta' \right) \end{aligned} \right\}. \quad (43)$$

Similarly, when the reference vector is locating in triangle II, the on-times for the vectors  $V_b$ ,  $V_c$  and  $V_a$  can be calculated by

$$\left. \begin{aligned} t_{\alpha'+1,\beta'} &= T_s \left( (V_{ab}/V_{dc}) + \beta' + 1 \right) \\ t_{\alpha',\beta'+1} &= T_s \left( (V_{ca}/V_{dc}) + \alpha' + 1 \right) \\ t_{\alpha'+1,\beta'+1} &= T_s \left( (V_{bc}/V_{dc}) - \alpha' - \beta' - 1 \right) \end{aligned} \right\}. \quad (44)$$

As an example, consider that the amplitude of reference vector is  $1.386V_{dc}$  and the angle is  $25^\circ$  same as used in previous algorithms. So, the line voltages remain same and the value of  $V_{ra'}$  and  $V_{rb'}$  is given by

$$\begin{bmatrix} V_{ra'} \\ V_{rb'} \end{bmatrix} = V_{dc} \begin{bmatrix} 1.380 \\ -0.795 \end{bmatrix}. \quad (45)$$

The exact location of reference vector can be determined by the sign of (40).

$$W = V_{bc} - \alpha' - \beta' - 1 = 0.585 - 1 + 1 - 1 = -0.415 \quad (46)$$

The sign of the above expression is negative so the reference vector is located in triangle 3 in Fig. 9 and the nearest three vectors are  $V_1(1, -1)$ ,  $V_2(1, 0)$  and  $V_8(2, -1)$ . The duty cycle for these vectors is calculated by (43).

$$\left. \begin{aligned} t_1 &= 0.415T_s \\ t_2 &= 0.205T_s \\ t_8 &= 0.380T_s \end{aligned} \right\} \quad (47)$$

#### D. Switching-State Selection

The last step of the algorithm is to transform the two-dimensional system back to three-dimensional switching states by using the following expression:

$$\begin{bmatrix} S_A \\ S_B \\ S_C \end{bmatrix} = \begin{bmatrix} i \\ i + \beta' \\ i - \alpha' \end{bmatrix} \in (0, 1, 2), \quad i \in (0, 1, 2). \quad (48)$$

For example, the vectors (0, -1) and (-1, 2) in the Fig. 9 can be transformed into (1 0 1), (2 1 2) and (0 2 1) switching states respectively.

## VI. SVPWM BASED ON IMAGINARY COORDINATE SYSTEM

The space vector diagram for three-level inverter based on imaginary coordinate system is shown in Fig. 11. Three axis of

imaginary coordinate system  $ja$ ,  $jb$  and  $jc$  are perpendicular to the three-phase axis  $a$ ,  $b$  and  $c$  respectively [52].

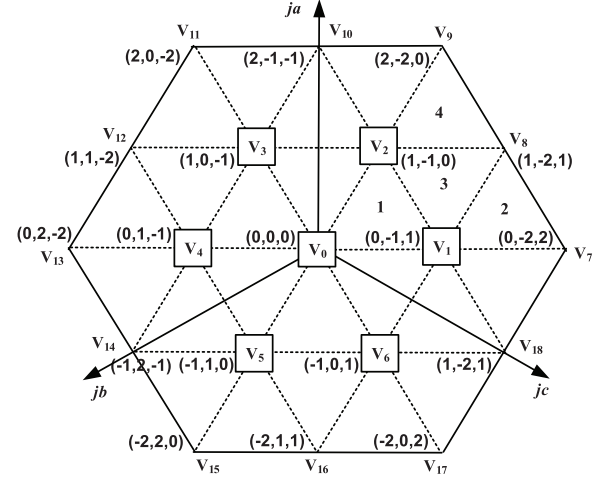


Fig. 11. Space vector diagram of inverter in  $ja$ - $jb$ - $jc$  frame.

#### A. Coordinate Transformation

The transformation from phase coordinate to the imaginary coordinate is given by

$$\begin{bmatrix} V_{ja} \\ V_{jb} \\ V_{jc} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (49)$$

where  $V_{ja}$ ,  $V_{jb}$ ,  $V_{jc}$  are variables of imaginary coordinate system and  $V_a$ ,  $V_b$ ,  $V_c$  are variables in phase coordinate system. The relation between imaginary frame and  $\alpha$ - $\beta$  frame is given by

$$\begin{bmatrix} V_{ja} \\ V_{jb} \\ V_{jc} \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{2} \\ -\sqrt{3/2} & -1/\sqrt{2} \\ \sqrt{3/2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}. \quad (50)$$

#### B. Identification of Nearest Three Vectors

All the vectors in imaginary coordinate system have integer values as shown in Fig. 11.  $\text{int}()$  is a function that is used to rounded down the real values to the integer values:  $V_{ja} = \text{int}(V_{ja})$ ,  $V_{jb} = \text{int}(V_{jb})$ ,  $V_{jc} = \text{int}(V_{jc})$ . There are two possible directions of equilateral triangles as shown in Fig. 12. If  $\text{int}(V_{ja}) + \text{int}(V_{jb}) + \text{int}(V_{jc}) = -1$ , then the reference vector is locating in normal direction triangle and if  $\text{int}(V_{ja}) + \text{int}(V_{jb}) + \text{int}(V_{jc}) = -2$ , then the reference vector is locating in reverse direction triangle.

#### C. Duty Cycle Calculation

Assuming that the reference vector is locating in normal direction triangle. The coordinates of three vertices are  $(V_{ja}+1, V_{jb},$



$V_{jc}-1$ ),  $(V_{ja}, V_{jb}+1, V_{jc}-1)$  and  $(V_{ja}, V_{jb}, V_{jc})$ . The time duration for these vertices is calculated by

$$\left. \begin{aligned} V_{ref} &= V_{ja}d_{ja} + V_{jb}d_{jb} + V_{jc}d_{jc} \\ d_{ja} + d_{jb} + d_{jc} &= 1 \end{aligned} \right\} \quad (51)$$

where  $d_{ja}=h_a$ ,  $d_{jb}=h_b$  and  $d_{jc}=h_c$  are the duty ratio for the vertices and  $T_s$  is the sampling time.  $h_a$ ,  $h_b$  and  $h_c$  represents the distance from the edges of triangle to the reference vector and computed by

$$\left. \begin{aligned} h_a &= V_{rja} - \text{int}(V_{rja}) \\ h_b &= V_{rjb} - \text{int}(V_{rjb}) \\ h_c &= V_{rjc} - \text{int}(V_{rjc}) \end{aligned} \right\} . \quad (52)$$

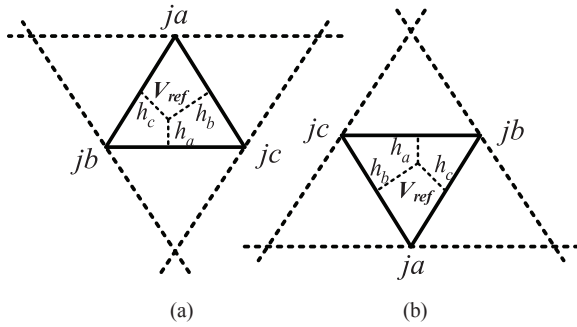


Fig. 12. Two possible directions of triangle. (a) normal direction and (b) reverse direction.

By using (51) and (52), the duty ratio for the corresponding vertices in line voltages form can be obtained by

$$\left. \begin{aligned} t_{ja} &= T_s \left( (V_{bc}/V_{dc}) - V_{ja} \right) \\ t_{jb} &= T_s \left( (V_{ca}/V_{dc}) - V_{jb} \right) \\ t_{jc} &= T_s \left( (V_{ab}/V_{dc}) - V_{jc} \right) \end{aligned} \right\} . \quad (53)$$

Similarly, when the reference vector is locating in reverse direction triangle, the duty ratio for the corresponding vertices is calculated by

$$\left. \begin{aligned} t_{ja} &= T_s \left( -(V_{bc}/V_{dc}) + V_{ja} + 1 \right) \\ t_{jb} &= T_s \left( -(V_{ca}/V_{dc}) + V_{jb} + 1 \right) \\ t_{jc} &= T_s \left( -(V_{ab}/V_{dc}) + V_{jc} + 1 \right) \end{aligned} \right\} . \quad (54)$$

For example, considering that the length and the angle of reference vector is  $1.386V_{dc}$  and  $25^\circ$  respectively. The line voltages are given by

$$\begin{bmatrix} V_{ja} \\ V_{jb} \\ V_{jc} \end{bmatrix} = \begin{bmatrix} V_{bc} \\ V_{ca} \\ V_{ab} \end{bmatrix} = V_{dc} \begin{bmatrix} 0.585 \\ -1.380 \\ 0.795 \end{bmatrix} . \quad (55)$$

The exact location of reference vector, whether it is located in

normal direction triangle or in reverse direction triangle is determined by the following expression:

$$\text{int}(V_{rja}) + \text{int}(V_{rjb}) + \text{int}(V_{rjc}) = 0 - 2 + 0 = -2 . \quad (56)$$

Because the value is -2, so the reference vector is located in reverse direction triangle and the nearest three vectors are  $V_1(0, -1, 1)$ ,  $V_2(1, -1, 0)$  and  $V_8(1, -2, 1)$ . The duty cycle for these vectors is calculated by (54).

$$\left. \begin{aligned} t_{ja} &= t_1 = 0.415T_s \\ t_{jc} &= t_2 = 0.205T_s \\ t_{jb} &= t_8 = 0.380T_s \end{aligned} \right\} \quad (57)$$

#### D. Switching-State Selection

The last step of the algorithm is to transform the two-dimensional system back to three-dimensional switching states by using the following expression:

$$\begin{bmatrix} S_A \\ S_B \\ S_C \end{bmatrix} = \begin{bmatrix} i \\ i - jc \\ i + jb \end{bmatrix} \in (0, 1, 2), \quad i \in (0, 1, 2) . \quad (58)$$

For example, the vectors  $(1, -1, 0)$  and  $(0, -2, 2)$  in the Fig. 11 can be transformed into  $(2, 2, 1)$ ,  $(1, 1, 0)$  and  $(2, 0, 0)$  switching states respectively.

It is clear that the SVPWM based on imaginary coordinate is the simplest way in duty ratio calculations.

## VII. CONCLUSION

This paper summarizes five different SVPWM algorithms for multilevel inverters which are  $\alpha$ - $\beta$  frame,  $g$ - $h$  frame,  $K$ - $L$  frame,  $\alpha'$ - $\beta'$  frame and SVPWM based on imaginary coordinate system. These algorithms are general and applicable to any level of inverter. Actually SVPWM is a modulation technique which is based on line voltage. Therefore, the complex computations of conventional SVPWM are converted into simplified line voltages form as shown in TABLE II and the duty cycles calculated for all other SVPWM algorithms are compared. As an example, for validation of the calculated results, the length and the angle of reference vector was kept constant for all the SVPWM algorithms and the on-time of the corresponding vectors is calculated. The comparison results validate all the SVPWM techniques, but the SVPWM based on imaginary coordinate is the simplest one in duty ratio calculations, easier to understand and provides a better control for zero-sequence component for any level of inverter.

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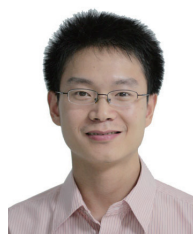
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